STABILITY AND $\aleph_0$-CATEGORICITY FOR NONASSOCIATIVE RINGS

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The model-theoretic properties of stability and $\aleph_0$-categoricity have been studied extensively in the context of groups and rings. Individually, each property induces a rich framework upon these algebraic objects; the joint presence of these properties severely restricts the algebraic structures one may encounter. Baur, Cherlin, and Macintyre [2] and, independently, Felgner [3] showed in the 1970s that a stable, $\aleph_0$-categorical group is nilpotent by finite. Baur, Cherlin, and Macintyre further conjectured that a stable, $\aleph_0$-categorical group is abelian by finite, a conjecture which remains open.

Contemporaneously, Baldwin and Rose [1] considered stable, $\aleph_0$-categorical associative rings and showed that they are nilpotent by finite. They further conjectured that such rings are null by finite, i.e. up to a finite ring extension, multiplication is trivial. This conjecture is equivalent to the conjecture on groups.

Rose [4] generalized many of the results on stable rings and $\aleph_0$-categorical rings to the nonassociative context of alternative rings. However, he was unable to show that a stable, $\aleph_0$-categorical alternative ring is nilpotent by finite. We have succeeded in proving this conjecture and we will outline the proof. We will also discuss the obstacles in generalizing this result to other nonassociative rings.

References

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[4] BRUCE I. ROSE, *Model theory of alternative rings*, **Notre Dame Journal of Formal Logic**, vol. 19 (1978), no. 2, pp. 215–243.